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СИНТЕЗ ОПТИМАЛЬНОГО УПРАВЛЕНИЯ ИНЕРЦИОННЫМИ ТЕХНОЛОГИЧЕСКИМИ ПРОЦЕССАМИ НА ОСНОВЕ МУЛЬТИАЛЬТЕРНАТИВНОГО ПАРАМЕТРИЧЕСКОГО ОПИСАНИЯ КОНЕЧНОГО СОСТОЯНИЯ

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СИНТЕЗ ОПТИМАЛЬНОГО УПРАВЛІННЯ ІНЕРЦІЙНИМИ ТЕХНОЛОГІЧНИМИ ПРОЦЕСАМИ НА ОСНОВІ МУЛЬТІАЛЬТЕРНАТИВНОГО ПАРАМЕТРИЧНОГО ОПИСУ КІНЦЕВОГО СТАНУ

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SYNTHESIS OF OPTIMAL CONTROL OVER INERTIAL TECHNOLOGICAL PROCESSES BASED ON A MULTI-ALTERNATIVE PARAMETRIC DESCRIPTION OF THE FINAL STATE

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Запропоновано метод пошуку оптимального за швидкодією і кінцевим станом управління технологічними процесами, заснований на ... Отримані при реалізації даного методу рішення повністю узгоджуються з результатами, що отримуються із застосуванням принципу максимуму Понтрягіна для задачі про швидкодію. Однак завдяки використанню в даному методі ... відкриваються додаткові можливості у вирішенні завдання ... Особливо слід відзначити, що при цьому спрощується процедура математичного конструювання оптимального управління технологічними процесами, що володіють властивостями істотної інерційності. Дані факти підтверджені результатами чисельного моделювання, які показали, що

Запропоновано та обґрунтовано два альтернативних варіанти реалізації управління, що відрізняються принципом вибору моментів перемикання управління. Показано, що визначальним фактором для вибору оптимального управління в цьому випадку є початковий стан системи щодо лінії кінцевого стану. Це зв'язано з тим, що ... Розроблено процедуру мультиальтернативного параметричного опису кінцевого стану, яка заснована на ... Такий підхід дозволяє ... Обґрунтуванням цього твердження може бути ... Показано, що в цьому випадку кожна з альтернатив є Завдяки цьому забезпечується можливість синтезу оптимального за швидкодією і кінцевим станом управління інерційними технологічними процесами в умовах мультиальтернативності опису кінцевого стану технологічної системи

Ключові слова: оптимальне керування технологічними процесами, принцип максимуму Понтрягіна, мультіальтернативний опис кінцевого стану,...

Предложен метод поиска оптимального по быстродействию и конечному состоянию управления технологическими процессами, основанный на ... Полученные при реализации данного метода решения полностью согласуются с результатами, получаемые с применением принципа максимума Понтрягина для задачи о быстродействии. Однако благодаря использованию в данном методе... открываются дополнительные возможности в решении задачи ... Особенно следует отметить, что при этом упрощается процедура математического конструирования оптимального управления технологическими процессами, обладающими свойствами существенной инерционности. Данные факты подтверждены результатами численного моделирования, показавшими что

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Предложены и обоснованы два альтернативных варианта реализации управления, отличающиеся принципом выбора моментов переключения управления. Показано, что определяющим фактором для выбора оптимального управления в этом случае является начальное состояние системы относительно линии конечного состояния. Это связано с тем, что ...

Разработана процедура мультіальтернативного параметрического описания конечного состояния, основанная на Такой подход позволяет...

Обоснованием этого утверждения может быть... Показано, что в этом случае каждая из альтернатив представляет собой Благодаря этому обеспечивается возможность синтеза оптимального по быстродействию и конечному состоянию управления инерционными технологическими процессами в условиях мультіальтернативности описания конечного состояния технологической системы

Ключевые слова: оптимальное управление технологическими процессами, принцип максимума Понтрягина, мультіальтернативное описание конечного состояния, ...

A method is proposed to search for control over technological processes, optimal in terms of speed and final state, on the basis of The solutions, obtained during implementation of the given method, are completely consistent with the results obtained using the Pontryagin maximum principle for the speed problem. Moreover, employing ... in this method opens up additional opportunities in solving the task ... It should be emphasized that in this case the procedure for mathematical construction of optimal control over technological processes that possess the properties of essential inertiality is simplified. These facts are confirmed by the results of numerical modeling, which showed that...

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Two alternatives of the control implementation are proposed and justified that differ in the principle of selecting control switching times. It has been shown that the

determining factor for the choice of optimal control is the initial state of the system, relative to the final state line. *This is linked to ...*

A procedure for the multialternative parametric description of the final state is developed, based on... Such an approach enables ... A substantiation of such argument is ... It is shown that in this case each of the alternatives represents ... Due to this, it is possible to synthesize the optimal control in terms of the speed and final state of sluggish technological processes under conditions of a multi-alternative description of the final state of the technological system.

Keywords: *optimal control over technological processes, Pontryagin's maximum principle, multi-alternative description of the final state*

1. Introduction

The search for optimal control over specific technical applications requires a substantiated choice of the methods that would be most suitable for these objects. When dealing with management of industrial objects and corresponding technological processes, it is necessary to take into account that such a choice should help cope with a number of uncertainties.....

In this regard, one interesting fact cannot be overlooked. Despite... the classical methods ... are very much in demand This is confirmed by studies devoted to [1–3] ... The solutions obtained by using these methods make it possible to ... provided ... All the aforementioned arguments substantiate the relevance of research subject devoted to ...

2. Literature review and problem statement

In study [4], it is argued that the search for optimal control makes it necessary to take into account ... In this case, development of the control object's model is considered as ... It is clear that the structure of such a functional reflects the interrelations, processes, characteristics and parameters of energy-efficient and management nature. Along with developing the criterion of effectiveness of resource use as a component of the criterion of management optimality, it allows identifying ... However, the description of functional proposed in the given paper does not take into account ... This leads to the fact that when trying to apply it to control inertial objects, there are objective difficulties. They are related, in particular, to ... This imposes certain constraints on the application of solutions proposed in [4]. The description of such constraints can be found in paper [5] that addresses, in particular, accounting for inertiality. The approach to identifying the object of control, employed in it, is based on the construction of an adaptive algorithm for calculating ... Applying this algorithm makes it possible to overcome the constraints imposed on ... That is why the solutions, obtained in paper [5], allow the possibility to ... However, despite the benefits of such algorithm, there is still an open question about ... It is of special relevance for the class of objects that ...

Development of views related to modeling such objects can be found in article [6], which demonstrated that ... In spite of the fact that such a problem was detected properly, while the recommendations regarding ..., the mathematical apparatus and algorithms for its implementation were given in article [6] as concepts only. The

author of the article confines himself to providing a conditional scheme of interaction with a standard TP ACS, as well as a general representation of the mathematical apparatus and the information-management system itself.

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 Systematization of results of the research conducted makes it possible to argue that the existing approaches to solving a problem are based on ... In other words, it is assumed that; in this case, this conclusion is valid also for ... It is obvious that similar approaches allow finding acceptable solutions only provided the finite state of the process is known with a preset degree of accuracy, while its mathematical description takes the form ... Hence it follows that a violation of the given conditions will not make it possible to obtain optimal solutions for the control systems over objects ... The specified part of the problem can be solved by developing a procedure for the synthesis of control over technological processes, optimal by criteria, which is based on receiving a multi-alternative description of the finite state.

3. The aim and objectives of the study

The aim of the study is to investigate the possibility of synthesizing the optimal control for the speed and final state of technological processes in the case of alternatives in describing the final state of the system.

To achieve this aim, it is necessary to solve the following tasks:

- to check the possibility of finding the optimal control, relying only on the analysis of the system of differential equations describing a mathematical model of the control object;
- to develop an algorithm for describing the final state of the system – controlled technological system of differential equations that describes a mathematical model of the process of control over the object.
- to propose a scheme for the practical implementation of the obtained solutions based on standard information-management systems in the structure of ACS over inertial technological processes.

4. Investigation of the solutions to a system of differential equations describing a mathematical model of a control object

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 To answer the question about the possibility of obtaining a simpler solution, it is advisable to consider a simple system of the type

$$\dot{x}_1(t) = x_2(t), \dot{x}_2(t) = u(t), t \in [0, T], |u(t)| \leq 1, \quad (1)$$

where $x_1(t)$ is the coordinate of the material point, $x_2(t)$ is the velocity of the displacement of the material point, and $u(t)$ is the control.

The SDE of form (1) is a mathematical model of the control object for both the initial conditions $x_1(0) = x_0, x_2(0) = \dot{x}_0$ and the final state, $x_1(T) = x_2(T) = 0, T \rightarrow \min, \dots$ the problem of finding optimal control is known as the problem on damping the

material point [37]. Its solution using the Pontryagin maximum principle is shown in Fig. 1.

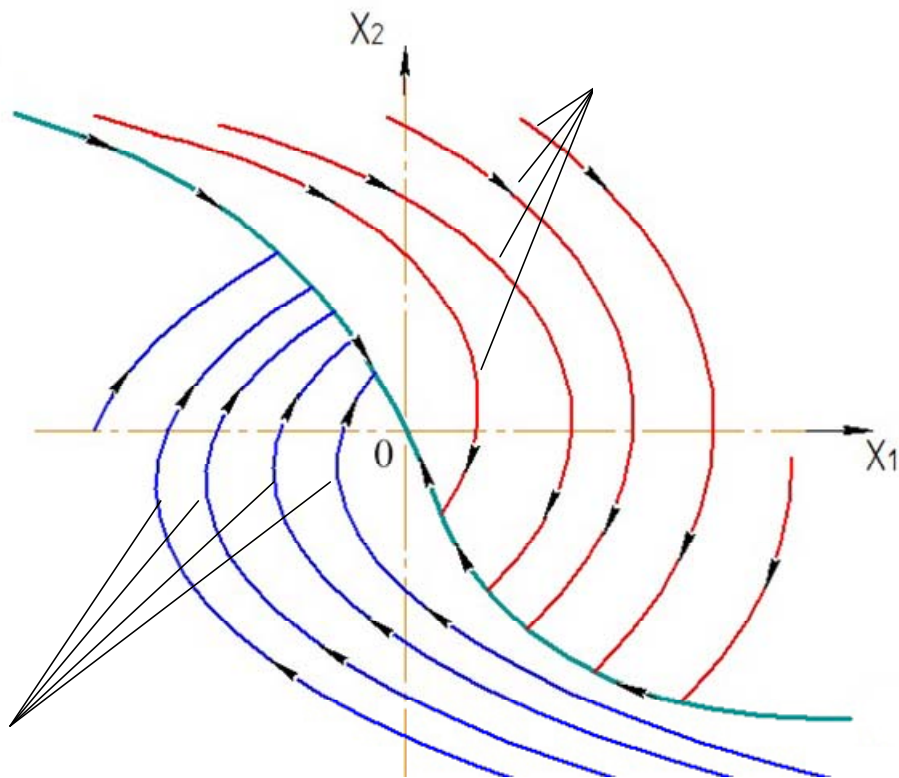


Fig. 1. Solving the problem of damping a material point using the Pontryagin maximum principle: $u_{opt} = \text{sign}(C_1 t - C_2)$, $u_{opt}(t)$ is the optimal control; C_1 , C_2 , C_3 , and C_4 are the integration constants

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Based on the obtained results of the synthesis of control for the two options, the latter can be compared and a choice can be made in favor of the one that provides the best indicators for stabilizing the object. The latter can be estimated, for example, on the basis of the time of the object (technological system) stay within the permissible area of the final state.

5. Synthesis of a multialternative description of the final state of a system in the problem of searching for optimal control

The choice of the final state is uniquely dependent on the task being solved and, in relation to the management of technological processes, the requirements imposed in specific conditions must be taken into account. The presence of a qualitative mathematical model of the control object is decisive; therefore, special attention is paid to problems of mathematical modeling for specific technological problems. A number of such applied studies can be found in the following:

– articles [41, 42], devoted to modeling in the search for optimal control of drilling processes;

– papers [43–45], devoted to modeling and managing in technologies of growing crystals;

– works [46–48], devoted to the modeling and identification of controlled objects using the analysis of interval sets and the study of the convergence of the solution of an extreme problem under constraints.

All of them are united by the general idea that the modeling and management of real objects involve the overcoming of objective difficulties caused by a fuzzy description and the need to take into account the significant, often multilevel, uncertainty. Some generalizations of such problems can be found in article [49], and general principles for their solution are described in paper [50].

In the event that an adequate mathematical model of the technological process is unknown, it can be obtained in at least two ways. The first of them is based on the implementation of an active experiment as a result of which the values of the coefficients of the regression equation describing the influence of input variables on output variables are estimated. The subsequent carrying out of the procedures of experimental optimization makes it possible to obtain a description of the stationary area on the basis of calculating the corresponding coefficients of the regression equation a_i [51]:

$$a_i = c_1 \sum_{j=1}^N x^j y^j, \quad i = 1, \dots, n, \quad (11)$$

$$a_i = c_2 [(x_i^j)^2 - \bar{x}_i^2] y^j, \quad i = n+1, \dots, 2n, \quad (12)$$

$$a_i = c_3 \sum_{j=1}^N x^j x^j y^j, \quad i = 1, \dots, n, \quad , \quad i = 2n+1, \dots, k, \quad (13)$$

$$a_0 = \frac{1}{N} \sum_{j=1}^N y^j - \sum_{j=1}^N a_{n+i}. \quad (14)$$

In formulae (11)–(14), c_1 , c_2 , and c_3 are the coefficients for linear, quadratic and paired relationships, respectively; n is the number of linear terms of the equations; N is the number of experiments; β is a parameter calculated depending on the number of points in the core of the compositional plan 2^{n-p} , the arm of the “star” points α and the number of points in the plan according to formula

$$\frac{\sum_{j=1}^N (x_i^j)^2}{N} = \frac{2^{n-p}}{N}. \quad (15)$$

An illustration of the solution to the optimization problem in the parametric form (9) and the derivation on the basis of its multialternative description of the final state is shown in Fig. 11–13. Fig. 13 also shows the principle of choosing the final state with respect to which optimal control should be sought.

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As a final state, it is proposed to use the parametric description of the form

$$\begin{aligned} x^*(t) &= (I - A)^{-1} a, \\ r(t) &= \sqrt{x^{*T} x^*}, \\ y^*(t) &= a_0 + 2a' x^* + x^{*T} A x^*, \end{aligned} \quad (25)$$

where a_0 , a , A the coefficients in the regression equation, $x_i^* = \frac{a_i}{2}$ are the suboptimal values of input variables – in the problems on searching optimal control over technological processes – state variables $r = \sqrt{r^2}$, $r^2 = \sum_{i=1}^n \frac{a_i^2}{2}$ are the limitations in factor space, $y^* = a_0 + \sum_{i=1}^n \frac{a_i^2}{2}$ are the suboptimal values of the output variable.

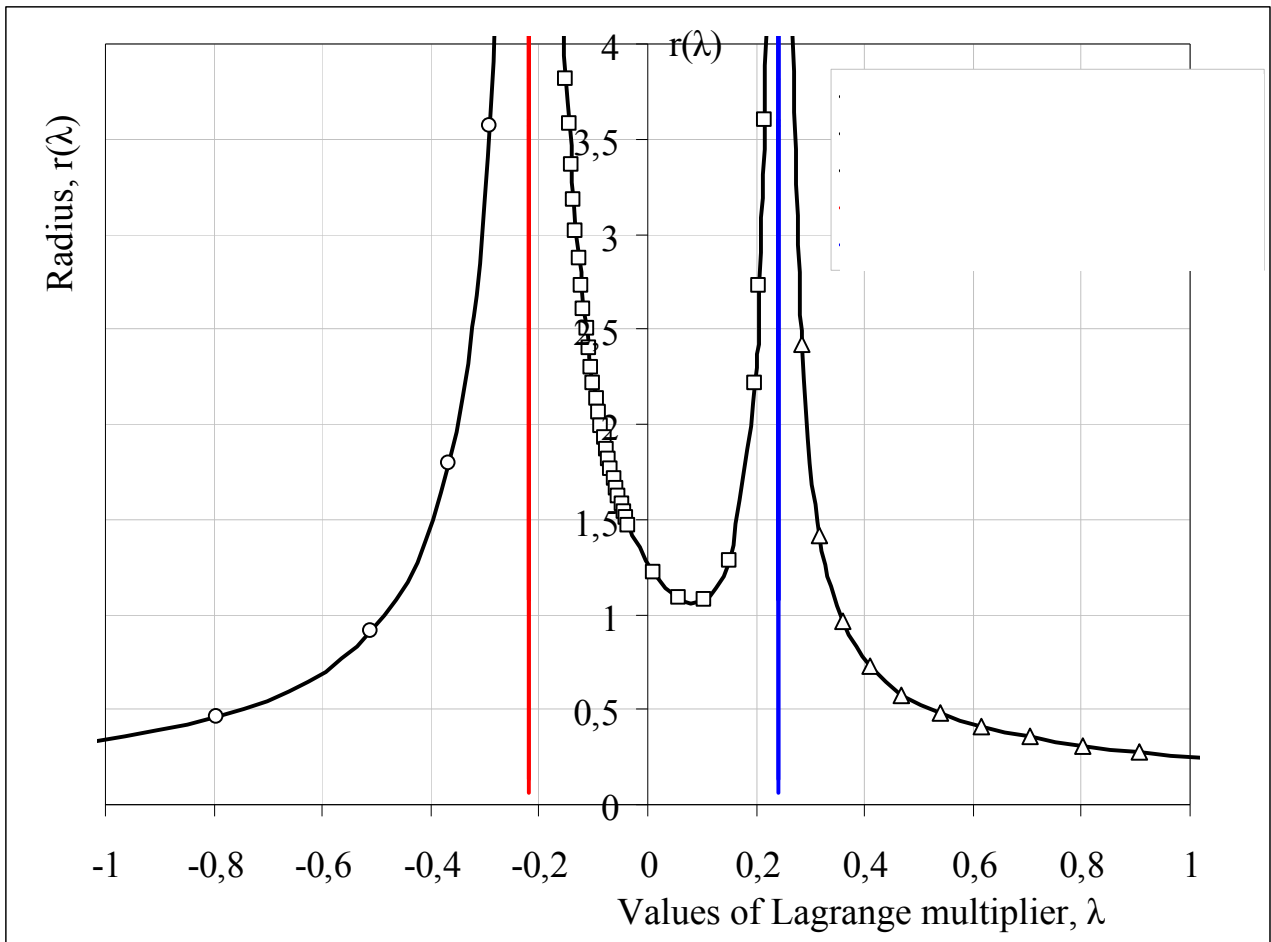


Fig. 11. Parametric description of limitations

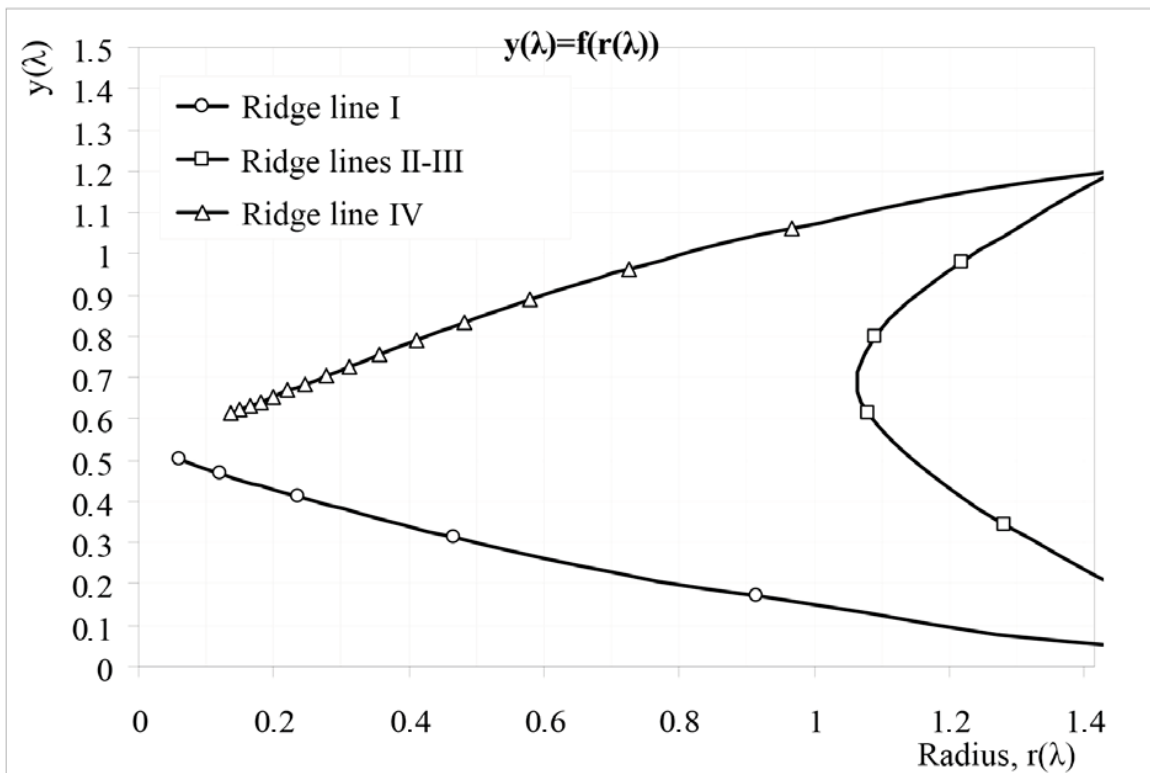


Fig. 12. Line of ridges – the curves describing a set of sub-optimal values

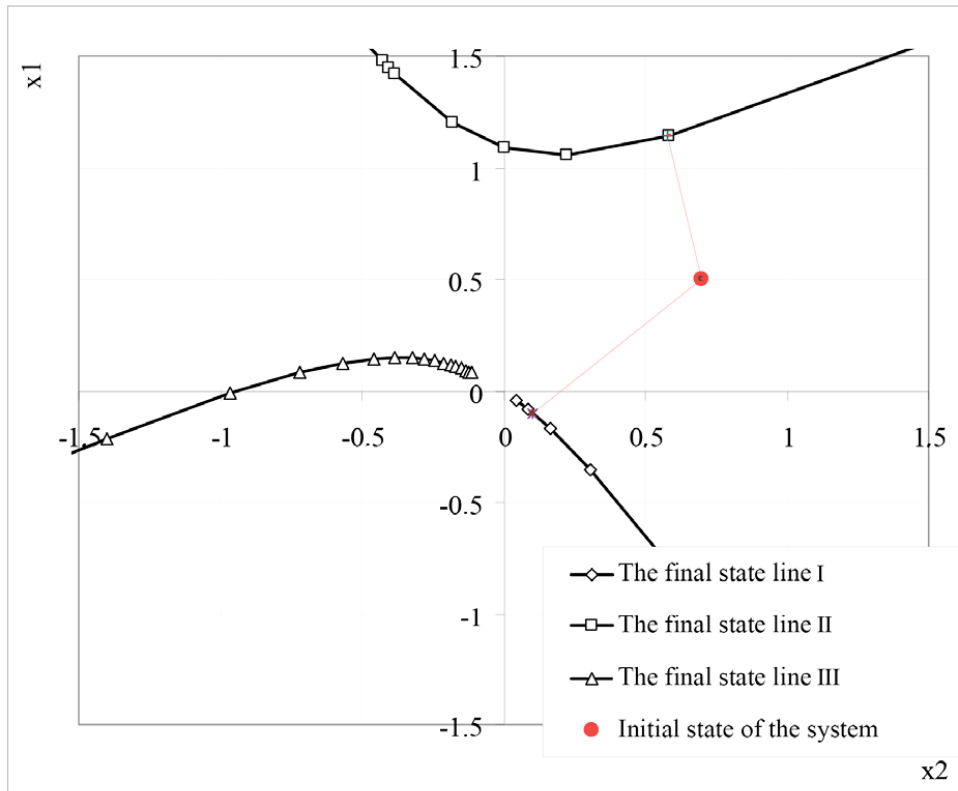


Fig. 13. Principle of a multialternative description of the final state and the principle of choosing the final state with respect to which it is expedient to search for optimal control

Obviously, the search for optimal control should be conducted relative to the nearest final state line. Moreover, if the line is straight, the distance can be determined in the known way

$$d_{ij} = \sqrt{(x_{1j} - x_{10})^2 + (x_{2j} - x_{20})^2}, \quad (10)$$

where x_{1j} and x_{2j} are the coordinates of the points on the line of the final state, while x_{10} and x_{20} are the coordinates of the point describing the initial state.

Results of numerical modeling are given in Table 3.

Table 3

Results of numerical modeling of the optimal control object trajectory at initial conditions... and the presence of a two-alternative description of the final state

The algorithm for a procedure of the synthesis of optimal control over a technological process during implementation of the proposed procedure for a multi-alternative description of the final state is shown in Fig. 4.

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Fig. 14. Algorithm for a procedure of the synthesis of optimal control at the proposed multi-alternative description of the final state

6. Integration of the obtained solutions into a two-level TP ACS

The task of software technological implementation of TP ACS based on the results of synthesis of optimal control under conditions of a multi-alternative final state can be solved on the basis of a distributed two-level control system.

At the lower level, the system resolves the task ... and includes the following:

...

At the upper level, the task is solved ... For its implementation, the recommendations, given in paper [48], can be applied. In particular,

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Hardware part of the information-management system consists of:

.....

Figure 15 shows an example of the structural diagram of a distributed two-level control system over a technological process.

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Fig. 15. Structural diagram of a distributed two-level control system over a technological process

As it follows from the proposed circuit solution, there is a principal possibility of improving reliability ... This is predetermined by the application of ...

7. Discussion of the results of studying the optimal control synthesis method based on the analysis of the mathematical model of the control object and the multialternative description of the final state

The obtained results of numerical modeling (Figs 11–13) testify to the fact that that the proposed procedure for the synthesis of a multi-alternative description of the final state makes it possible to ... It is clear from Fig. 12 that the search for suboptimal solutions is based on determining the intersection point of curves describing ... Deriving the latter is carried out by solving a system of equations (25) and thus presents no difficulties. In this case, it should be emphasized that efficiency of the solutions, in a sense of ..., depends on ... This is particularly clear from Figs 4, 5 – the curves are mostly in the region of negative values of input variables ... In turn, hence it follows that the obtained result is explained by eliminating a contradiction between the need to achieve high performance and productivity. The latter is related to solving a problem on the compromise optimization due to employing a method ... when obtaining a multi-alternative description of the final state. is exactly why the proposed variant ... should be considered promising.

The proposed method for searching for the optimal control over technological processes, based on ..., has a benefit in that it ...

.....

It should be noted that the proposed method of searching for optimal control, based ... does not take into account such an important criterion of management quality as minimizing energy costs. This leads to ... That is why the given circumstance indicates an obvious shortcoming of the proposed method. The direction for further research, related to its elimination, should be aimed at developing a procedure for the adaptation of a dynamic programming method. Such adaptation should enable a possibility to impose additional constraints on the variables of state and controlling influences.

8. Conclusions

1. The proposed method of searching for the optimal speed and final state of control of technological processes, based on ... makes it possible, along with its simplicity, to obtain solutions completely consistent with the results obtained using the Pontryagin maximum principle. In doing so, it opens up additional opportunities for solving the problem on ... It has been shown that for this there are at least two alternative control implementations that differ The determining factor for choosing the optimal control in this case is the initial state of the system, described by the position of the phase space point characterizing the actual initial state relative to the final state line.

2. It is proposed to obtain a multi-alternative description of the final state using a method of ridge analysis. Obtaining the suboptimal solutions during its implementation allows ... A special feature of the given approach is that it eliminates the need for ... Such feature of the method makes it possible to simplify the procedure of mathematical construction of optimal control over technological processes that possess the properties of essential inertiality. It becomes possible due to the application of ... in the proposed method. The confirmation of the given fact can be found in the results of numerical modeling, which revealed ...

3. The proposed method for searching for the optimal control could be implemented in a standard scheme of information-management systems in the structure of TP ACS. For this purpose, it is necessary to employ a module ... that enables the operation ... Such module can be included in the circuit of software-hardware complex based on ... High performance efficiency is ensured by ... while a decrease in inertiality is provided by ... A signature of such circuit solutions is accounting for the components at the upper level ...

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